needs to be considered to reproduce the observed transmittance spectrum:

$$g(r) = \frac{w}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(r-r_1)^2}{2\sigma^2}\right] + \frac{1-w}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(r-r_2)^2}{2\sigma^2}\right], \tag{1}$$

where σ is the width of the distributions, and w and $1-w$ are the weights of the first and second size, respectively. We denote by c_{tot} the total number of spheres per unit volume; then $c_{tot}g(r)\mathrm{d}r$ is the number density of spheres with the size in the range $(r, r+\mathrm{d}r)$. The effective permittivity ε of a composite containing such a distribution of spheres with the polarizabilities $\alpha(r)$ reads:

$$\frac{\varepsilon-\varepsilon_H}{\varepsilon+2\varepsilon_H} = \frac{1}{3\varepsilon_0\varepsilon_H}\int_0^\infty c_{tot}g(r)\alpha(r)\mathrm{d}r. \tag{2}$$

The polarizability $\alpha(r)$ is obtained from the Mie theory [24, 25] as $\alpha(r) = 4\pi\varepsilon_0\varepsilon_H\, 3ia_1(r)/(2k^3)$, where $k = 2\pi f\sqrt{\varepsilon_H}/c$ is the wave vector of radiation in the host medium and a_1 is the electric-dipole Mie coefficient of a sphere with radius r. Similar equations can be derived also for the magnetic response where the magnetic-dipole Mie coefficient b_1 is introduced. We finally find:

$$\varepsilon = \varepsilon_H \frac{k^3 + 4\pi i c_{tot}\int_0^\infty g(r)a_1(r)\mathrm{d}r}{k^3 - 2\pi i c_{tot}\int_0^\infty g(r)a_1(r)\mathrm{d}r} \qquad \mu = \frac{k^3 + 4\pi i c_{tot}\int_0^\infty g(r)b_1(r)\mathrm{d}r}{k^3 - 2\pi i c_{tot}\int_0^\infty g(r)b_1(r)\mathrm{d}r}. \tag{3}$$

The use of the Maxwell-Garnett theory is well justified, because the concentration of microspheres is low (below 5%) and the spatial distribution is random. In the fits the transmission T was calculated using the standard Airy formula including all internal reflections.

A good match between the calculated and measured spectra is found for mean particle radii $r_1 = 17\ \mu\mathrm{m}$, $r_2 = 13.5\ \mu\mathrm{m}$ and a distribution width $\sigma = 1\ \mu\mathrm{m}$. These parameters are supported by a optical microscope analysis, which revealed a comparable ellipticity of the microparticles. Furthermore, the filling factors obtained from the fit by the Maxwell-Garnett theory (0.4%, 1.5% and 6.7%) are in good agreement with those determined from the weights of the components when preparing the pellets (0.4%, 1.0% and 5%, respectively). Note that in the Maxwell-Garnett theory calculations, we optimized the thickness of the pellets (typically within 30 μm) in order to match the interference pattern at low frequencies.

6. Conclusion

We fabricated rigid metamaterials made of TiO_2 spherical microresonators embedded in polyethylene. We observed a magnetic effective response in the vicinity of the first Mie resonance. Using finite-difference time-domain calculations of the effective response we found that a range of negative effective magnetic permeability can be achieved for sufficiently high filling factors and contrasts between the permittivities of the resonators and the embedding medium. The developed structures are prototypes of cheap mechanically stable terahertz metamaterials.

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